Angular Acceleration from Slope of Angular Velocity vs. Time Graph

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Introduction

When a figure skater does a sit spin, she *rotates* about a point at a certain number of degrees per second. When she pulls her arms in, her rate of rotation increases. This rate of rotation is known as angular velocity. Similarly, when a planet *revolves* around the sun, a line connecting the planet to the sun sweeps out a certain number of degrees per unit of time. In this case, the rate of revolution of the planet about the sun is known as its angular velocity. These two examples illustrate that there are two kinds of angular velocity—*spin* angular velocity and *orbital* angular velocity. In both cases, the angular velocity could be expressed in degrees/second.

Now let's imagine that the intelino smart engine is moving around the circular track of Figure 1 at a constant linear speed in cm/s. A line connecting the engine to the center of the circle sweeps out an angle θ in *t* seconds. Then the orbital angular velocity of the engine would be θ/*t* degrees/second. While traveling around the track, the same side of the engine always faces the center of the circle. As a result, the engine spins once around its center in the same time that it makes one revolution. Therefore, the engines spin angular velocity would be the same as its orbital angular velocity, *i.e.*, θ/*t* degrees/second. This is analogous to the revolution of the moon around the earth, where the moon keeps the same side facing the earth at all times.

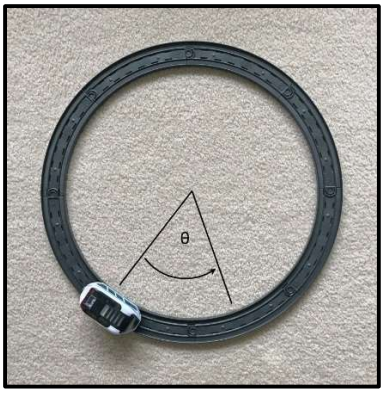


Figure 1

In this lesson, we will be concerned only with the orbital angular velocity of the engine. We'll simply call this *angular velocity* for short, and we will measure this in ^{o}/s, *i.e.*, degrees/second. If the linear speed of the engine increases over time, the angular velocity will also increase. The

rate at which angular velocity increases is known as *angular acceleration*. The unit of measure for angular acceleration of the intelino engine will be ^{o}/s/s or $^{o}/s^2$, i.e., degrees/second-squared. For example, it the engine increased its angular velocity from 20 ^{o}/s to 60 ^{o}/s in a period of 8 seconds, its angular acceleration would be (60 – 20) ^{o}/s divided by 8 s, or 5 ^{o}/s/s.

Objectives

- Collect data on the engine's angular velocity while its linear speed gradually increases.
- Using a spreadsheet, create a graph of angular velocity versus time.
- Compute the angular acceleration from the slope of the angular velocity graph.

Materials

- Intelino engine
- 8 curve tracks
- 2-8 white, 2-8 magenta color action snaps
- Intelino extension for Scratch 3.0
- Scratch link program (free download)
- Windows 10 (versions 1709+) or macOS (versions 10.13+) with Bluetooth
- *Angular Velocity and Acceleration.sb3* Scratch program (provided with this lesson)
- Spreadsheet program

Suggested Track Layout

Figure 2 shows the suggested track layout for performing this lesson. Eight curve tracks are joined to form a circle. Four color action snaps (white-magenta) are placed 90^{o} apart. The engine is placed on the track with the forward direction shown by the arrow. Each time the engine encounters an action snap, two things will happen. The Scratch timer and current angular velocity will be recorded to two Scratch lists. The linear speed of the engine will be increased by an amount specified by the student.



Figure 2

Working with the "Angular Velocity and Acceleration.sb3" Scratch Program

Although the Scratch program contains a fair number of blocks, the only blocks that you need to be concerned with are shown in Figure 3. The blocks let you set four parameters: degrees between snaps, speed increment per snap, starting nominal speed, and maximum nominal speed. Comments for each of the blocks provide additional detail.

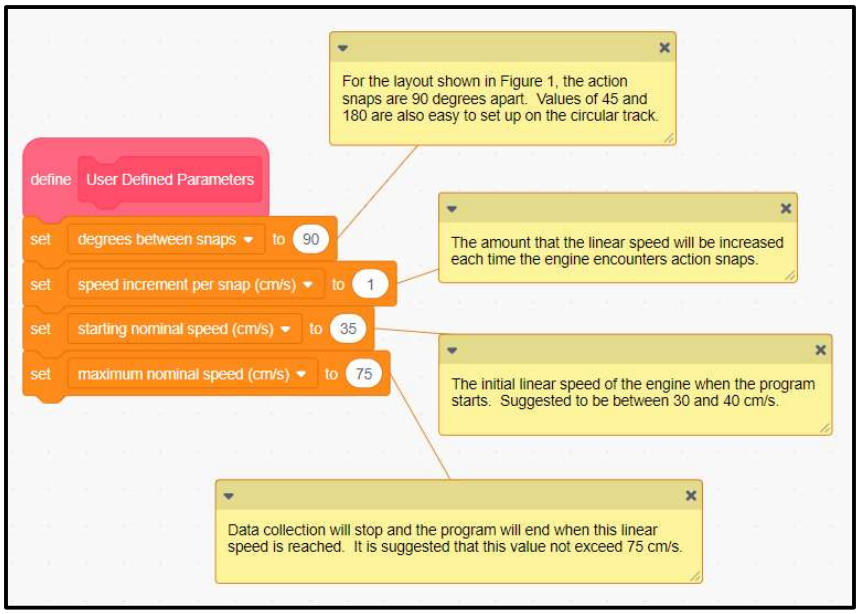


Figure 3

Procedure

1. Construct the track layout of Figure 2 with color action snaps 90^{o} as shown.
2. Leave the user defined parameters at the default values shown in Figure 3.
3. Place the intelino engine in the position shown in Figure 2.
4. Load the *Angular Velocity and Acceleration.sb3* Scratch program.
5. Start the program by clicking on this icon in the upper right corner of the Scratch screen:
   
6. Intelino will run for at most 1 minute, with its linear speed increasing at each of the action snaps. Increase in angular velocity will be quite noticeable.
7. You will see accumulated statistics in two lists in the upper right area of the Scratch screen. Figure 4 shows typical lists at the conclusion of the program run. The "Time" list contains values for the Scratch internal timer as data collection progresses. The "Angular Velocity" list contains the current value for angular velocity of the engine at each of the recorded times.
8. Right click on each list to export the data in each list. Then copy the exported data to your favorite spreadsheet program.

9. Repeat data collection with “speed increment per snap” values of 0.5 cm/s and 2 cm/s. Export these results to your spreadsheet.
10. Determine the value of the slope of the line for each of the three “speed increment values per snap” for which you collected data: 0.5, 1, and 2 cm/s. You can do this either by (1) using a ruler to eye a straight line of good fit through the data points, or (2) using the spreadsheet’s trendline analysis feature with a linear fit. In the latter case, the slope will be the coefficient of the independent variable time.
11. What is the unit of measure for the slope values? Is this unit of measure that for angular acceleration discussed earlier in this document.
12. What do you predict the angular acceleration will be if the “speed increment per snap” is set to 0.25 cm/s? Check out your prediction by performing this scenario with intelino and the provided Scratch program.
13. What do you conclude about the relationship between the slope of an angular velocity versus time graph and the value of the angular acceleration?

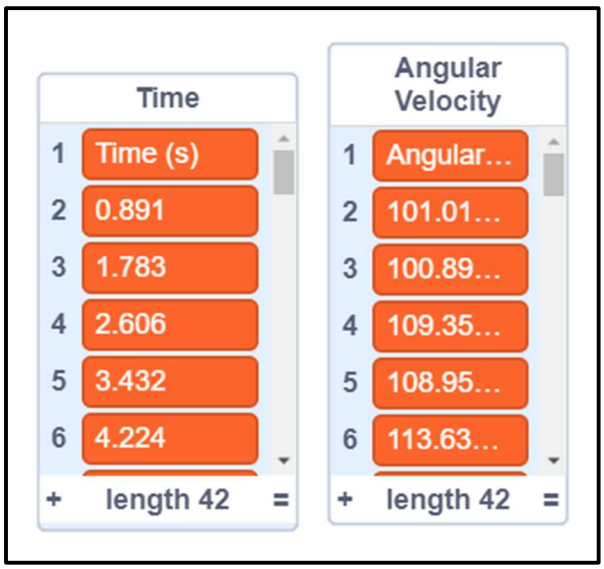


Figure 4