Constant Acceleration

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Introduction

A measure of automotive performance of a car in the United States and United Kingdom is the time it takes to accelerate from 0 to 60 mph. In the rest of the world, 0 to 100 km/h is the standard, corresponding to 0 to 62.1 mph.

For example, the 2020 Chevy Corvette can accelerate from 0 to 60 mph in 2.9 seconds. This is an average acceleration of about 20.7 mi/hr/s! The unit of measure for acceleration always involves one distance unit and two time units. The two time units can be the same. As an example, it would be quite common to describe the intelino Smart Train's acceleration in cm/s/s, or cm/s^2 when using the intelino Scratch extension.

Acceleration is a vector quantity, as it has both magnitude and direction. In this lesson, however, we will be concerned only with the magnitude of the acceleration in the direction of motion of the intelino Smart Train. This is sometimes referred to as tangential acceleration. On a curved track, this would be the rate of acceleration of the train tangential to the curved track, *i.e.*, the acceleration in the direction of motion.

Lesson Goal

In this lesson, you will learn two ways to determine the train's acceleration: (1) from the slope of its speed versus time graph, and (2) from a specific characteristic of its distance versus time graph. You will accomplish this by analyzing distance and speed data collected in the provided Scratch program, *Constant Acceleration.sb3*.

Track Layout

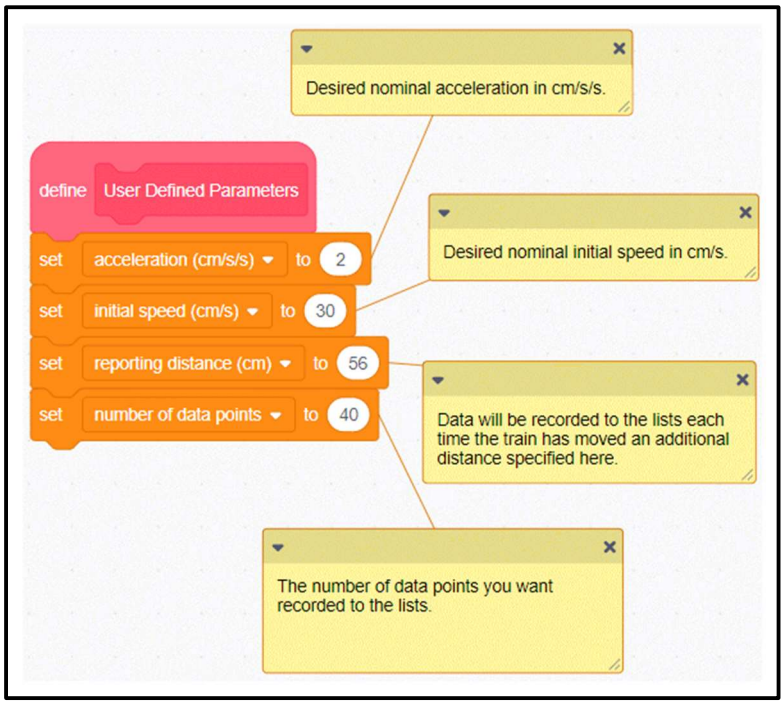
You can make use of the track layout shown below, which can be constructed with a single set of intelino tracks. Alternatively, you can construct your own layout with as many tracks as you wish. Just make sure that your layout is a closed loop **with no split tracks**. You don't want to use any split tracks because intelino's initial firmware has been designed to automatically speed up when turning left or right off of a split track, especially at low speeds. This speed increase provided more torque and momentum, helping with steering the train. It is possible that future firmware versions may eliminate the need for this speed-up feature.



Working with the Scratch Program

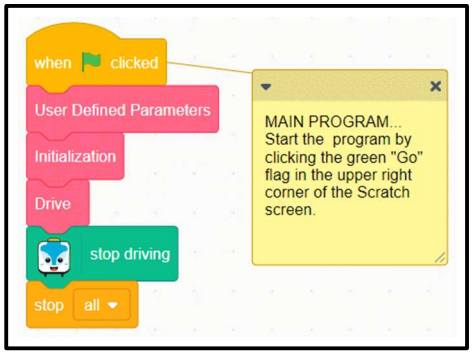
The only blocks in the Scratch program that you need to be concerned with are shown in the figure below. These blocks let you set four parameters:

1. *The desired nominal acceleration in cm/s/s*. The figure shows the setting as 2 cm/s/s.
2. *The initial speed in cm/s*. It is suggested that this *not be lower* than 30 cm/s.
3. *The reporting distance in cm*. Every time that the train has moved this additional distance, data on time, total distance, and speed are recorded in Scratch lists. Data from these lists can then be exported as *txt* files to Notepad. From there, the data can be copied to a spreadsheet program for further analysis. Note that in the figure, the reporting distance was set to 56 cm. To help visualize the reporting distance, the train's light will change colors each time the data are reported.
4. *Number of data points*. This is the number of data points that you want recorded to the lists. The train will stop and the Scratch program terminate after the last data point has been recorded.



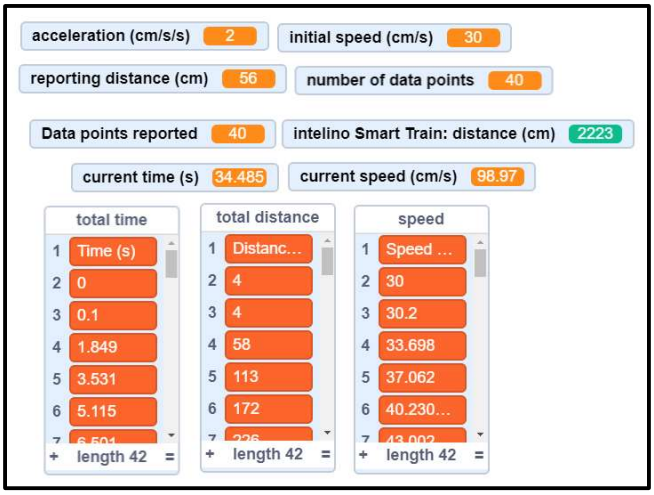
Starting the Scratch Program

The main program making up the Scratch program is shown in the figure below. The purpose in showing this figure is to let you know how easy it is to start the program. Place the Smart Train anywhere on the track layout, and click on the green "Go" flag in the upper right corner of the Scratch screen. We have already reviewed the *User Defined Parameters* block, and you need not be concerned with the details of the *Initialization* and *Drive* blocks. The stop driving block is a block found in the intelino Scratch extension and does what its name suggests. The stop all block stops the Scratch script.



Typical Statistics from the Scratch Program

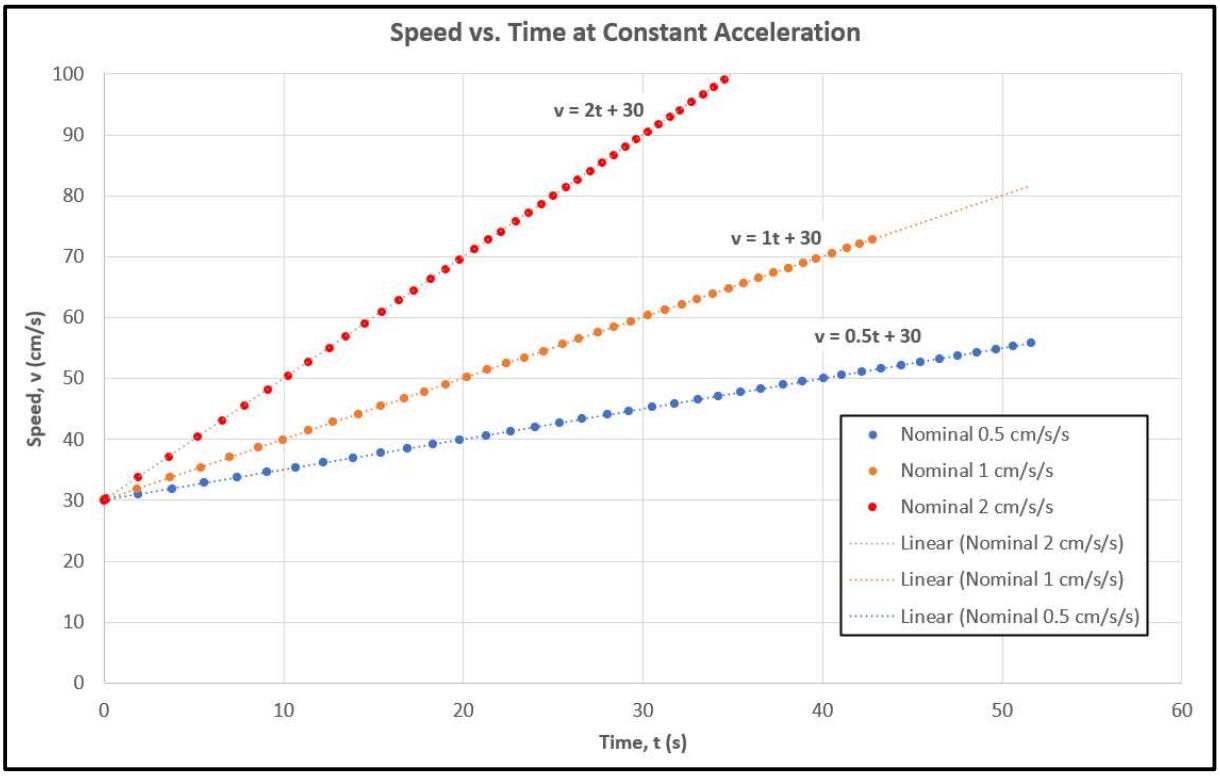
When the program has completed its run and the intelino Smart Train comes to a stop, you will see statistics similar to the following in the upper right Scratch window.



The values for the four user defined parameters—*acceleration, initial speed, reporting distance,* and *number of data points*—are at the top. The *data points reported*, *intelino Smart Train: distance: (cm)*, *current time (s)* and *current speed (cm/s)* are in the center. The three lists—*total time, total distance*, and *speed* are at the bottom.

Speed vs. Time Graph

The Excel graph of speed versus time shown below contains typical results with nominal accelerations of 0.5, 1, and 2 cm/s/s. Excel's 'trendline' feature was used for a linear fit to the data, with the equations of best fit shown on the graph. The slope increases with increasing acceleration, and in all three cases, the nominal acceleration is the coefficient of time (t). The equations support the idea that $v = at + v_o$. This is in agreement with the well-known equation $v = v_o + at$ for motion with constant acceleration. In conclusion, we can say that the value of the acceleration can be obtained from the slope of a speed vs. time graph.

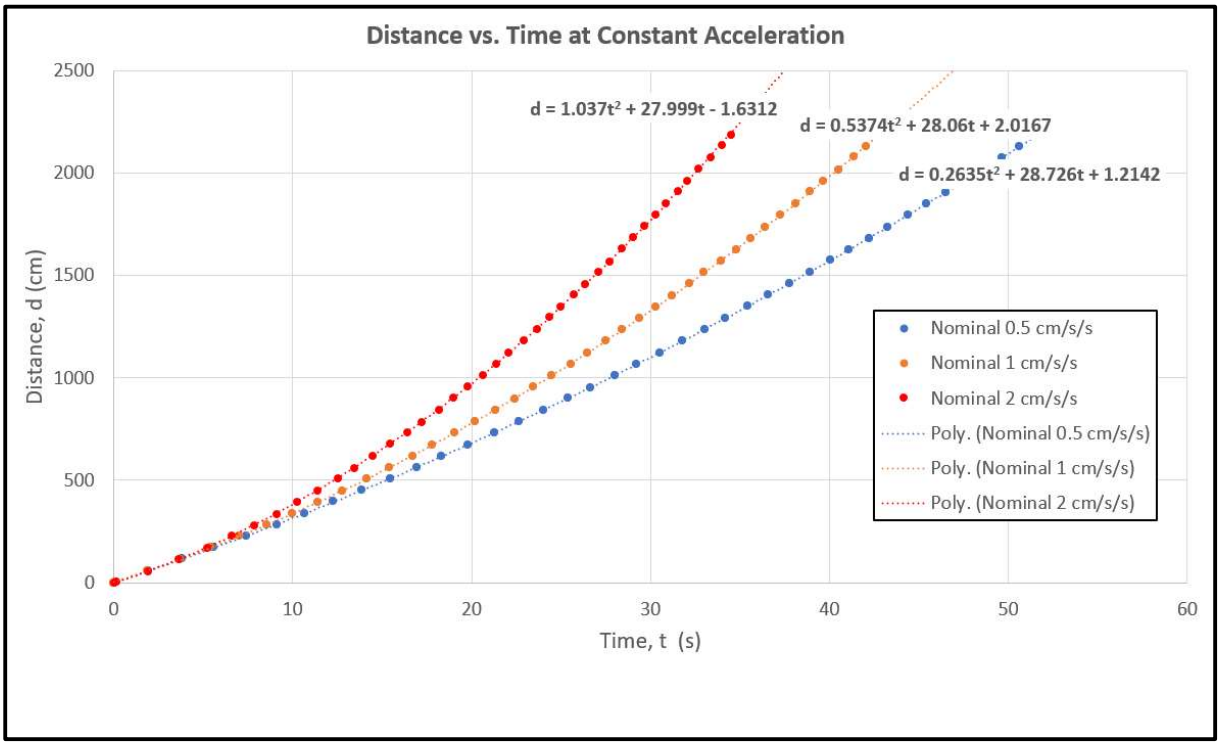


Distance vs. Time Graph

The Excel graph of distance versus time shown below contains typical results with nominal accelerations of 0.5, 1, and 2 cm/s/s. A visual inspection of the data curves shows that distance is *not* increasing linearly with time. The curves look like they could be parabolic in shape. Therefore, Excel's 'trendline' feature was used with a 2nd order polynomial (quadratic) fit to the data. The fit appears good and the equations of best fit are shown on the graph.

Since we know from the speed vs. time graphs that acceleration is constant, we are reminded of the well-known equation for motion with constant acceleration $d = \frac{1}{2}at^2 + v_o t + d_o$. The quadratic equations of best fit shown in the figure match this form. The constant d_o for all

three cases is very close to zero, meaning that the train started at the origin. (The origin is not too far from where you started the train on the track layout. To give time for the train to reach the specified initial speed, the train runs for 2 seconds prior to the start of any data collection.) The coefficient of t is the initial speed, all reasonably close to 30 cm/s. Most importantly, the coefficient of t^2 is $\frac{1}{2}a$. This means that we can find the acceleration by multiplying the coefficient of t^2 by 2. Therefore, for nominal accelerations of 0.5, 1, and 2 cm/s/s, our graphs indicate accelerations of 2•0.26 = 0.52, 2•0.54 = 1.08, and 2•1.04 = 2.08 cm/s/s, respectively. The agreement between the nominal values for acceleration and the experimental values is quite good.



Conclusion

Our original goal was to learn two ways to determine the train's acceleration: (1) from the slope of its speed versus time graph, and (2) from a specific characteristic of its distance versus time graph. Regarding (1), we saw that the acceleration can be found from the slope of the speed versus time graph. Regarding (2), we found that the acceleration can be found by doubling the coefficient of t^2 in the distance versus time graph.